MECHANICS OF SOLIDS (ME F211)

BITS Pilani
K K Birla Goa Campus

VIKAS CHAUDHARI

innovate achieve lead

Chapter-2

Introduction to Mechanics of deformable bodies

Introduction to Mechanics of deformable bodies

Analysis of deformable bodies

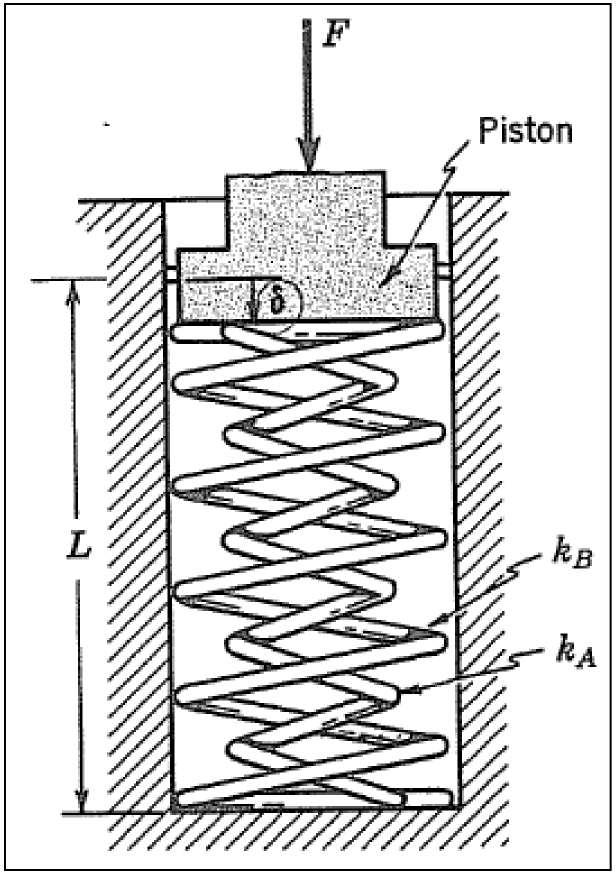
- Identification of a system
- Simplification of this system
- To develop model which can be analyzed

Steps to analyze system

- Study of forces and equilibrium requirements
- Study of deformation and conditions of geometric fit
- Application of force- deformation relations

Example:

A machine part carrying a load ***F*** terminates in a piston which fits into a cavity, as shown in Fig. Within the cavity are two linear springs arranged coaxial with each other. We use the symbols k_A and k_B to denote the spring constants of the two springs in the cavity. When the springs are unloaded, each has the same length *L*. We wish to know how much of the load ***F*** is carried by the spring with constant k_A



Solution

Steps to analyze the system

1. Selection of model of the actual system
2. Assumptions
3. Study of forces and equilibrium requirements
4. Study of deformation and conditions of geometric fit
5. Application of force- deformation relations to predict behavior of the system

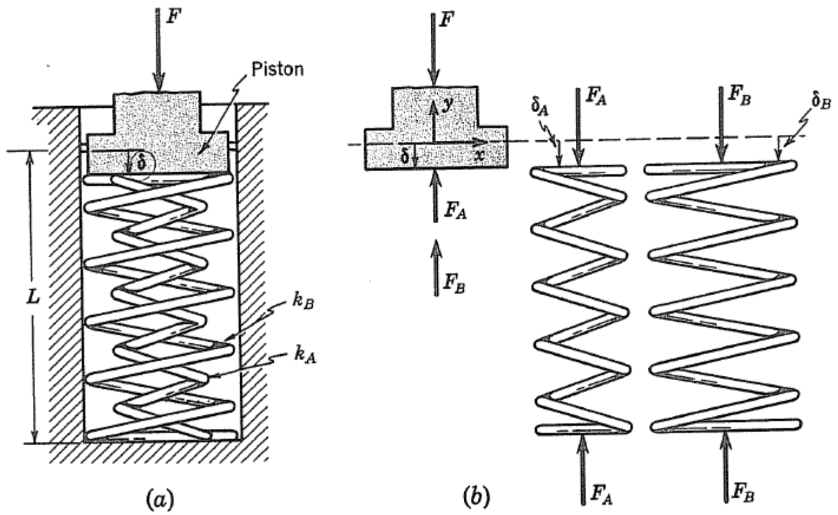
1. Selection of model
   a. Identify the elements of proposed model of the system. In given problem the system has two elements one Piston and two springs
2. Assumptions
   a. Assumed that the springs have been made with flat ends such that the compressing force which is distributed around the periphery of the spring, can be considered to act along the spring axis
   b. Assumed that gravity effects can be ignored
   c. Springs fit in the cavity perfectly without buckling

Introduction to Mechanics of deformable bodies

3. Study of forces and equilibrium requirements
   a. Isolate the elements from its surrounding i.e. Draw free body diagram
   b. Apply equations of equilibrium i.e. $\sum F = 0$ and $\sum M = 0$

$$\sum F_y = 0 = F_A + F_B - F \quad \text{--- 1}$$

Free body Diagram



Introduction to Mechanics of deformable bodies

4. Study of deformation and conditions of geometric fit

a. Find what are the requirements for geometric compatibility

$$\delta_A = \delta_B = \delta \qquad \text{--- } 2$$

5. Application of force- deformation relations

a. The force in each spring is linearly proportional to the deflection of the spring, and the constant of proportionality is the spring constant

$$F_A = k_A\, \delta_A \qquad \text{--- } 3$$

$$F_B = k_B\, \delta_B \qquad \text{--- } 4$$

Introduction to Mechanics of deformable bodies

By combining equation 1-4, we obtained desire result

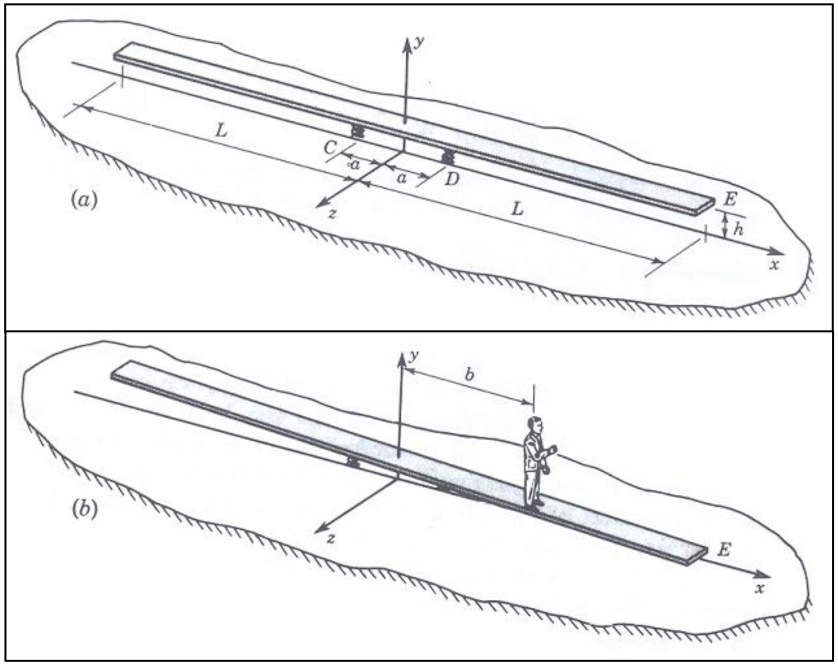
$$\frac{F_A}{F} = \frac{k_A}{k_A + k_B} \quad \text{and} \quad \frac{F_B}{F} = \frac{k_B}{k_A + k_B}$$

Total deflection of piston

$$\delta = \frac{F_A}{k_A} = \frac{F}{k_A + k_B} = \frac{F_B}{k_B}$$

Example:

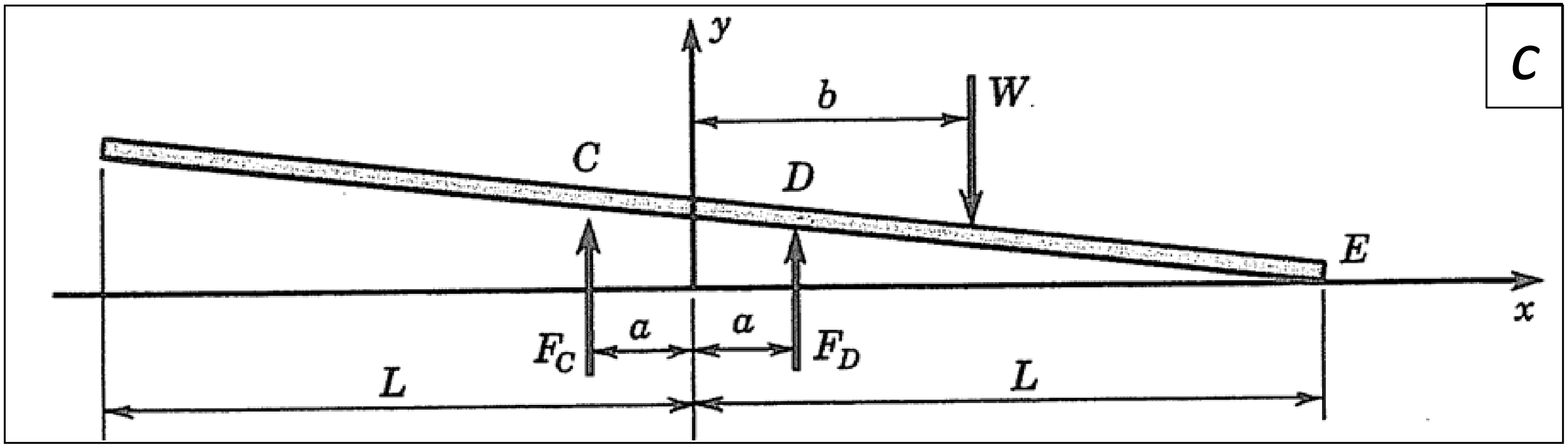
Let us consider a very light and stiff wood plank of length 2L. Two similar springs (spring constant k) are attached to it. The springs are of length h when the plank is resting on them. Let a man stands on middle of plank and begins to walk slowly towards one end. Find how far the man can walk before one end of the plank touches the ground



Solution:

1. Force equilibrium : Free body diagram

Reaction at point *E* will be ***zero*** since the plank is just touching the ground

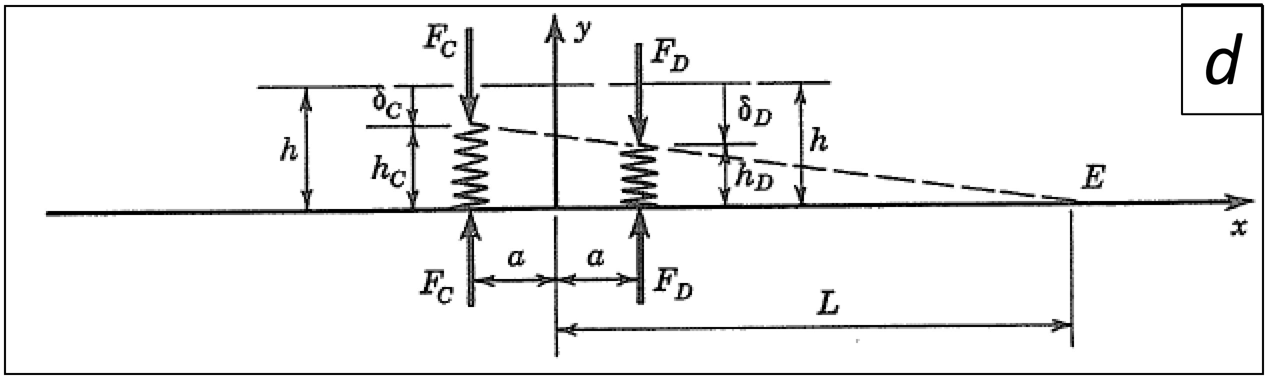


$$\sum F_y = 0 \quad \rightarrow \quad F_C + F_D - W = 0 \quad \text{--- 1}$$

$$\sum M_{@C} = 0 \quad \rightarrow \quad 2 \times a \times F_D - (a + b) \times W = 0 \quad \text{--- 2}$$

2. Study of deformation and conditions of geometric fit

When the plank remains straight, from the similar triangles



$$\frac{h_C}{h_D} = \frac{L+a}{L-a} \qquad \text{--- } 3$$

The deflections of the springs are

$$\delta_C = h - h_C \quad and \quad \delta_D = h - h_D \qquad \text{--- } 4$$

innovate achieve lead

3. Relations between forces and deflections

Since both springs are linear and have same spring constant,

$$F_C = k\delta_C \quad and \quad F_D = k\delta_D \qquad \text{--- 5}$$

Equation 1 to 5 will give seven independent relations for seven unknowns, F_C, F_D, h_C, h_D, δ_C, δ_D, and b.

Solving these equations, we find that the value of b is given by

$$b = \frac{a^2}{L}\left(\frac{2kh}{W} - 1\right) \qquad \text{--- 6}$$

Spring deflections

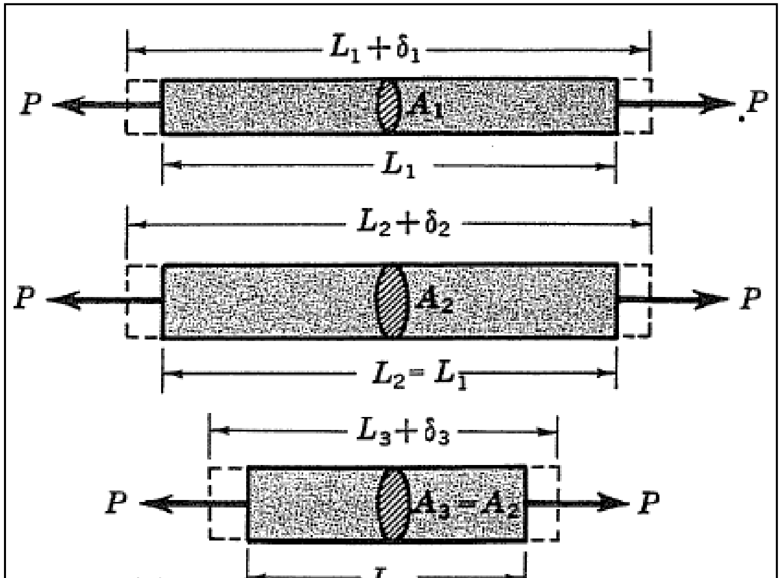
$$\delta_C = \frac{W}{2k}\left(1 - \frac{b}{a}\right) \quad \text{and} \quad \delta_D = \frac{W}{2k}\left(1 + \frac{b}{a}\right) \qquad \text{--- } 7$$

Highlights

- It can be seen that δ_D is always positive in the sense defined in Fig *d*.
- δ_C is positive as long as b < a. When b = a, the man is directly over the spring *D*, and, as would be expected, all the load is taken by the spring *D*, and the deflection and force in the spring *C* are zero.
- When b > a, then δ_C is negative (i.e., the spring extends). In Fig. *c* we assumed that b > a and that the spring *C* is compressed.

UNIAXIAL LOADING & DEFORMATION

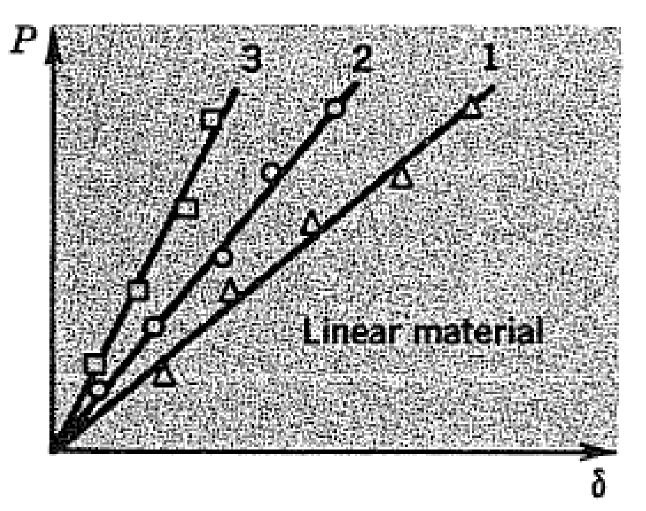
- The basic type of deformation which is considered in most of the problems in this chapter is shown in Figure (*a*).
- Consider the deformation of three rods of same material, but different lengths and cross-sectional areas as shown.
- Assume that for each bar the load is gradually increased from zero, and at several values of the load a elongation (δ) is measured.



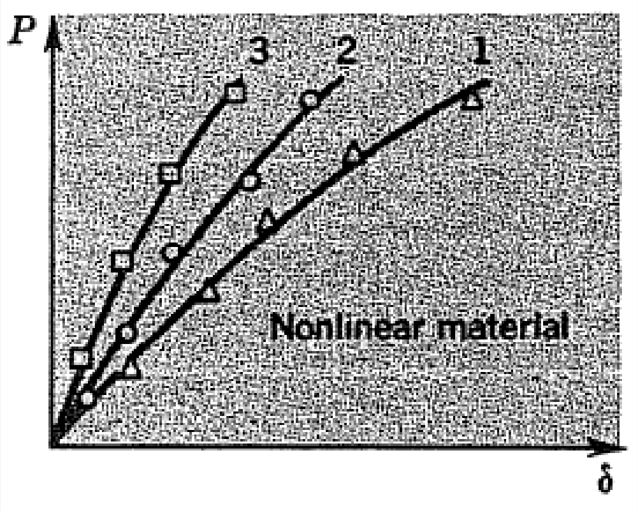
(a)

UNIAXIAL LOADING & DEFORMATION

- If the maximum elongation is very small (less than 0.1 percent of the original length), then for most materials the results of the three tests will be represented by a plot like Figure (*b*) or like Figure (*c*).



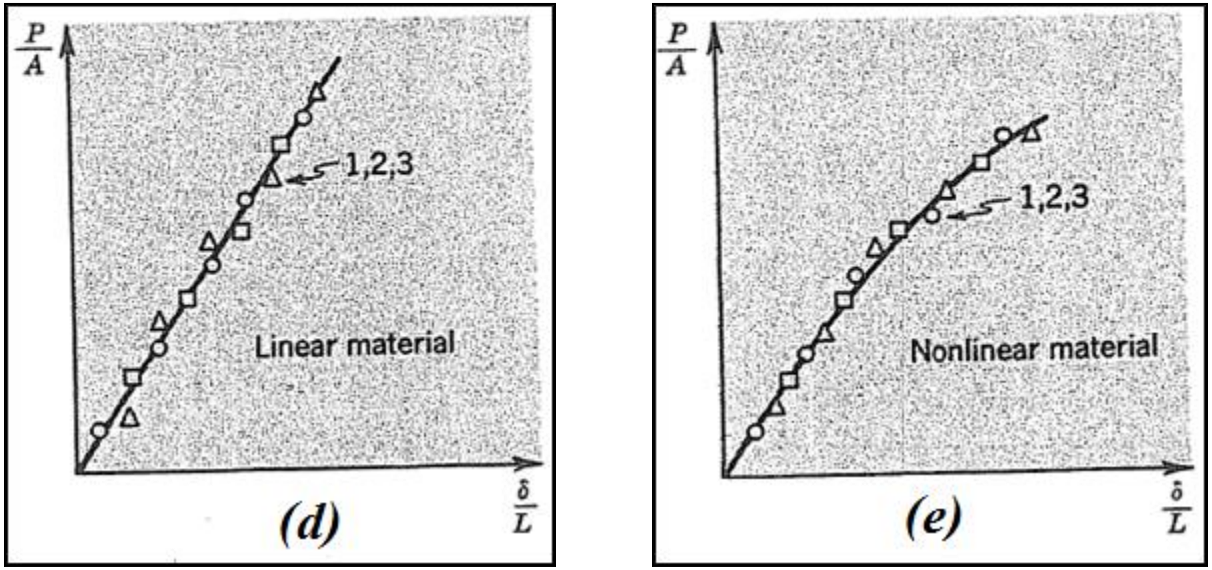
(*b*)



(*c*)

UNIAXIAL LOADING & DEFORMATION

- If the experimental data are re-plotted with load over area (*P/A*) as ordinate and elongation over original length (δ/L) as abscissa, the test results for the three bars can be represented by a single curve, as shown in Figure (*d*) or Figure (*e*).



UNIAXIAL LOADING & DEFORMATION

- If the uniaxial load-elongation relation of the material is linear as shown in Figure (*d*), then the slope of the curve is called the ***modulus of elasticity*** and is denoted by the symbol ***E***

$$E = \frac{P/A}{\delta/L} = \frac{PL}{\delta A}$$

- The dimension of ***E*** is N/m^2 or N/mm^2.
- Deflection can be given in terms of ***E***

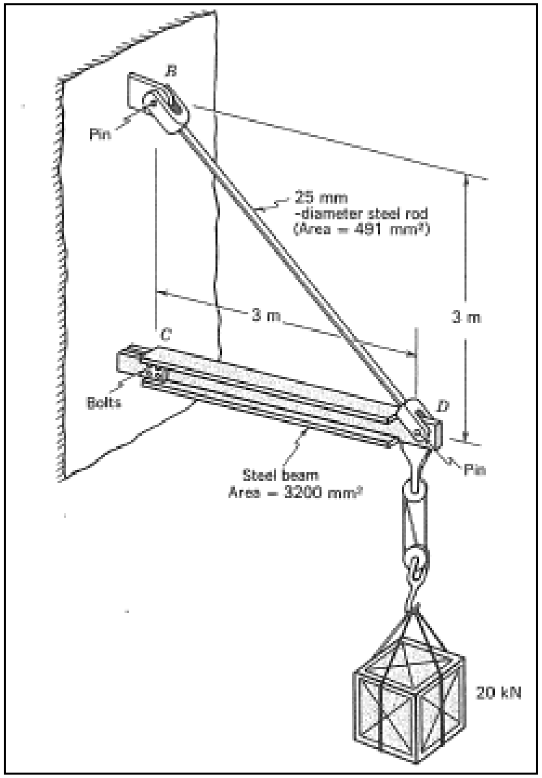
$$\delta = \frac{PL}{AE}$$

Statically determinate situations

These are the situations in which the forces can be obtained without reference to the geometry of deformation

Example

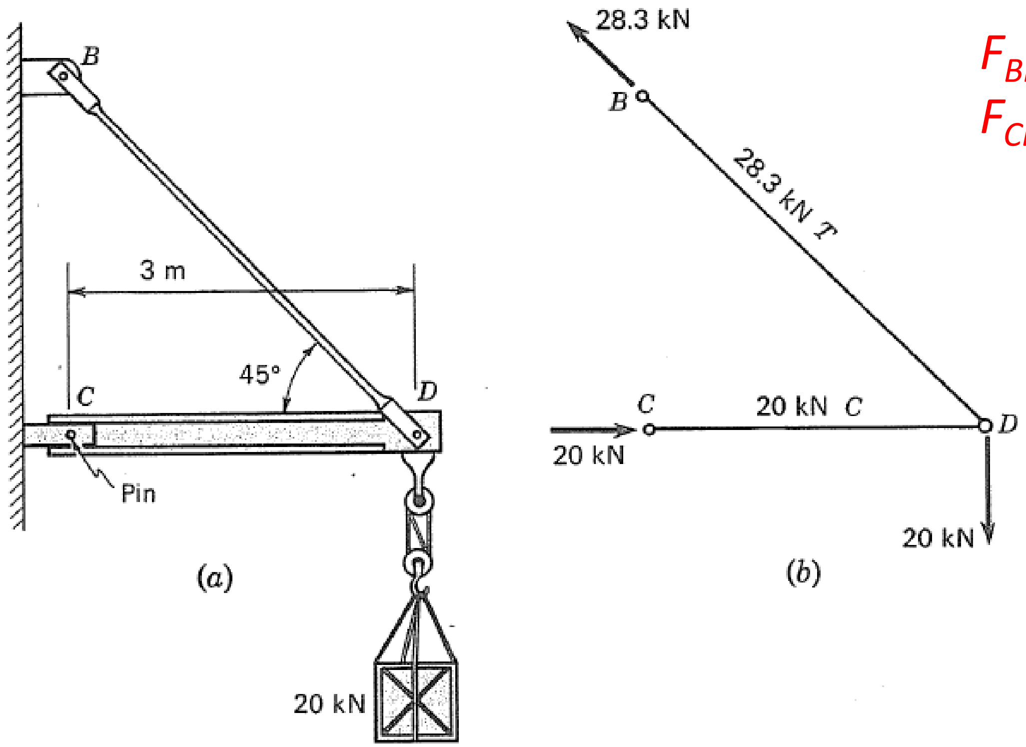
Figure shows a triangular frame supporting a load of 20kN. Our aim is to estimate the displacement at the point *D* due to the 20kN load carried by the chain hoist.



Solution

1. Force equilibrium:

   The forces in the members were determined in previous chapter



F_{BD} = 28.3kN Tensile

F_{CD} = 20kN Compressive

2. Force deformation relation

$$\delta_{BD} = \left(\frac{PL}{AE}\right)_{BD} = \frac{28.3\times10^3\times3\sqrt{2}\times10^3}{491\times205\times10^3} = 1.19mm \quad \text{--- extension}$$

$$\delta_{CD} = \left(\frac{PL}{AE}\right)_{CD} = \frac{20\times10^3\times3\times10^3}{3200\times205\times10^3} = 0.0915mm \quad \text{--- compression}$$

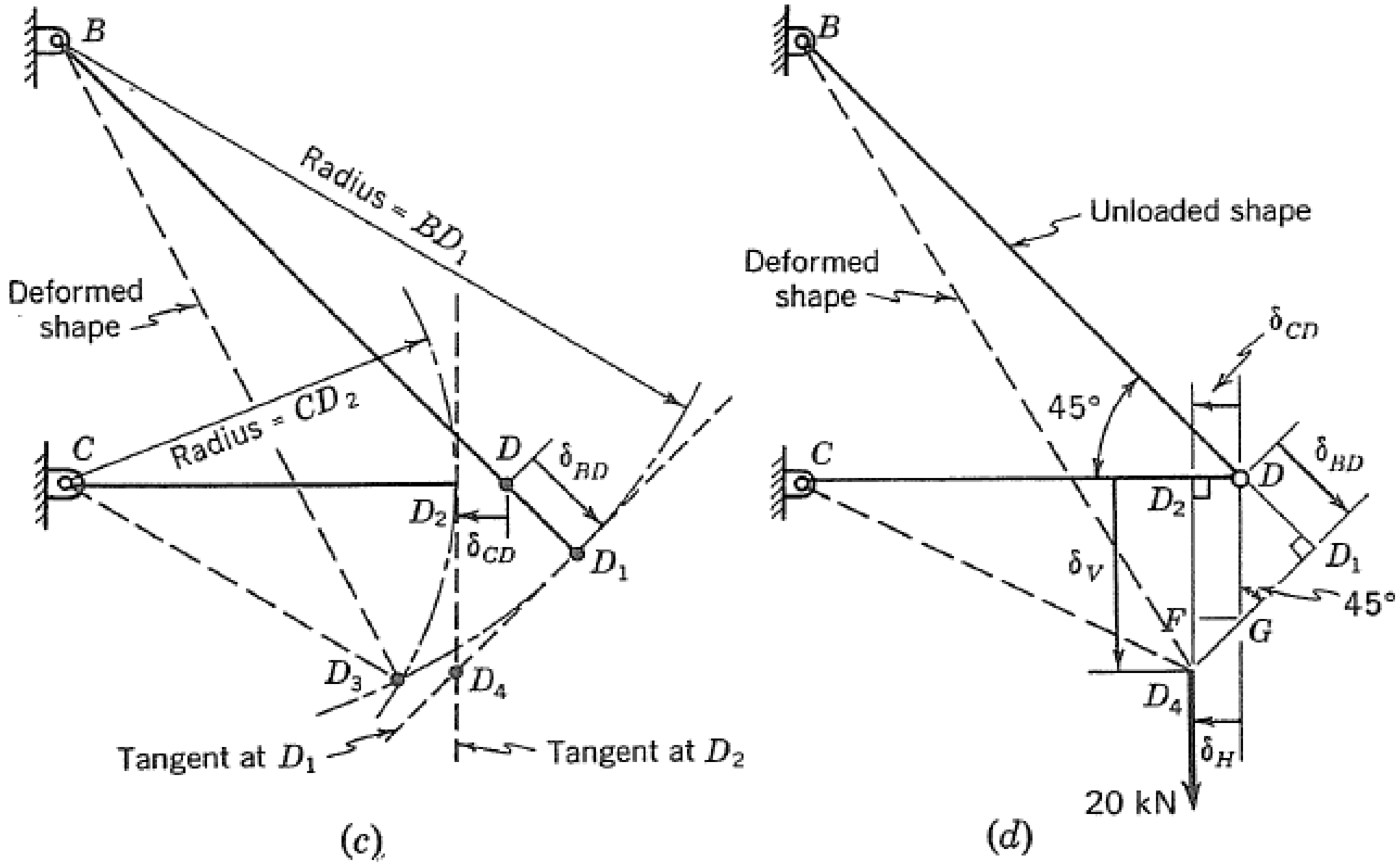
3. Geometric compatibility

- Bars *BD* and *CD* move in such a way that, after deformation, they remain straight and also remain fastened together at *D* as shown in Figure (*c*)
- Since the deformations of the bars are very small fractions of the lengths, we can replace the arcs by the tangents to the arcs at D_1 and D_2 and obtain the intersection D_4 as an approximation to the location of D_3 Fig (d).

Introduction to Mechanics of deformable bodies

$\delta_H = \delta_{CD} = 0.0915\text{mm}$

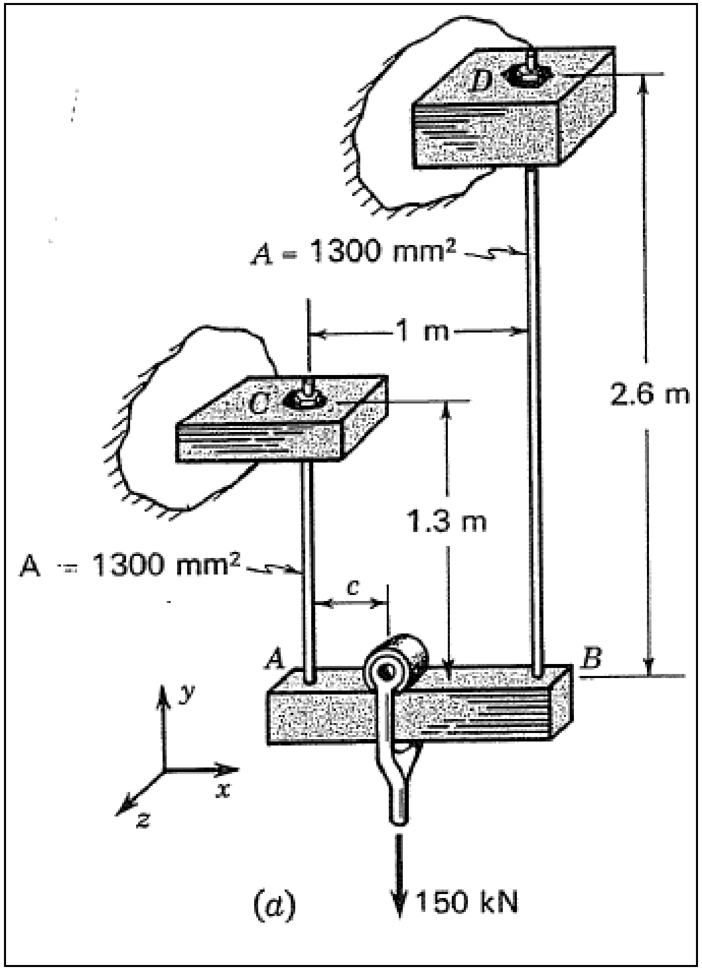
$\delta_V = D_2F + FD_4 = DG + FG = \sqrt{2} \times \delta_{BD} + \delta_{CD} = 1.77\text{mm}$

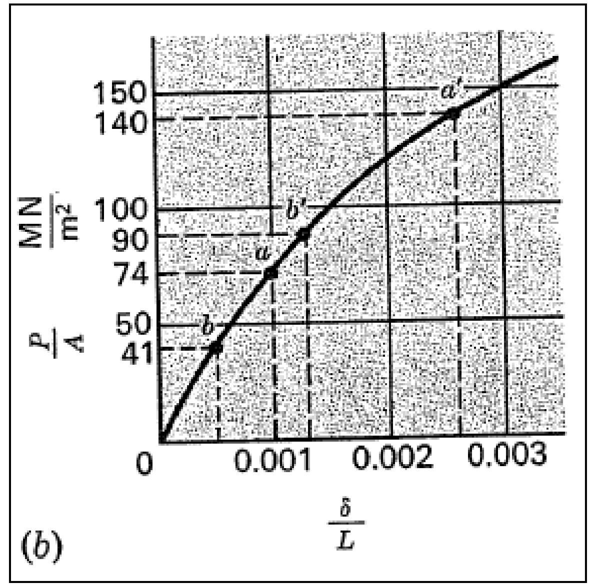


Example

The stiff horizontal beam *AD* in Figure (*a*) is supported by two soft copper rods *AC* and *BD* of the same cross-sectional area but of different lengths. The load-deformation diagram for the copper is shown In Figure (*b*). A vertical load of 150kN is to be suspended from a roller which rides on the horizontal beam. We do not want the roller to move after the load is put on, so we wish to find out where to locate the roller so that the beam will still be horizontal in the deflected position.

Introduction to Mechanics of deformable bodies





Introduction to Mechanics of deformable bodies

1. Force equilibrium

$$\Sigma F_y = 0 = F_A + F_B - 150 \qquad \text{--- 1}$$

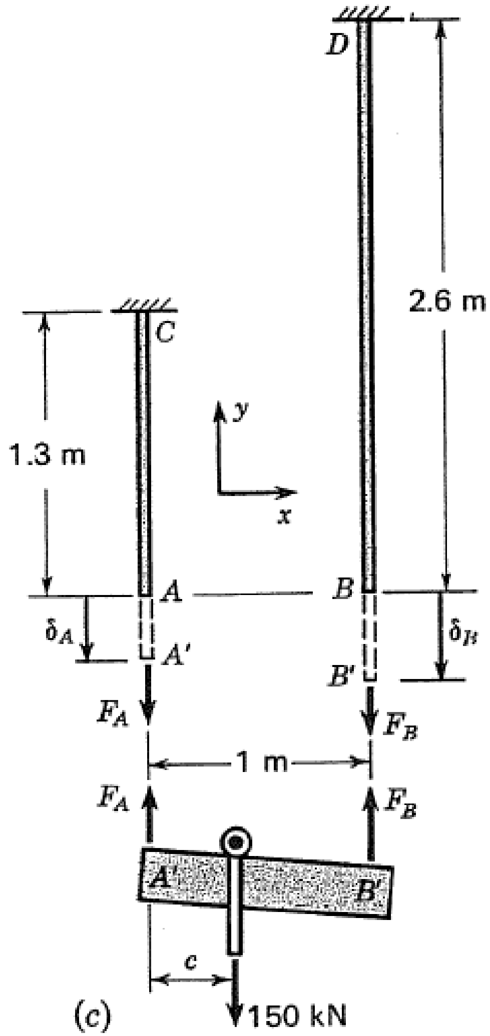
$$\Sigma M_{@A} = 0 = F_B \times 1 - 150 \times c \qquad \text{--- 1(A)}$$

2. Geometric compatibility

$$\delta_A = \delta_B \qquad \text{--- 2}$$

from the lengths of the bars

$$\frac{\delta_A}{L_A} = \frac{\delta_B}{L_A} = \frac{\delta_B}{1.3} = 2 \times \frac{\delta_B}{2.6} = 2 \times \frac{\delta_B}{L_B} \qquad \text{--- 3}$$

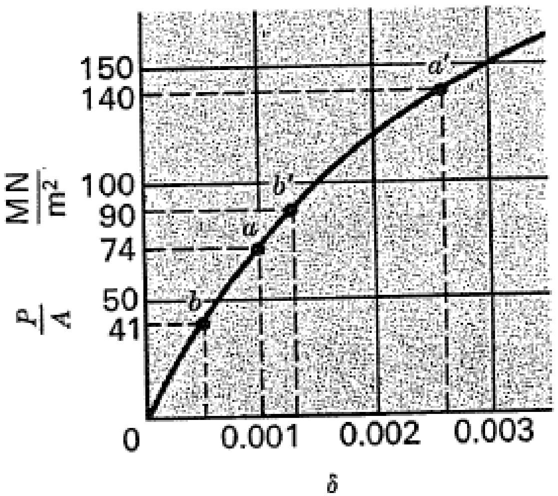


Introduction to Mechanics of deformable bodies

3. Force-deformation relation

In given problem, force and deformation are related with Figure (*b*).

e.g. if we choose δ_B/L_B in diagram equal to 0.001 then corresponding value of F_B/A_B would be 74 MN/m^2. --- 4



(*b*)

Equations 1, 3 and 4 represent our analysis of the physics of the problem. We must combine 1, 3, and 4 mathematically to find the correct location of the roller.

Introduction to Mechanics of deformable bodies

Dividing equation 1 by A_A and substituting $A_A = A_B = 1300\text{mm}^2$

$$\frac{F_A}{A_A} + \frac{F_B}{A_A} = \frac{150}{A_A} \quad \Rightarrow \quad \frac{F_A}{A_A} + \frac{F_B}{A_B} = 115\frac{MN}{m^2} \qquad \text{--- } 5$$

Procedure to proceed further

1. Select an arbitrary value of δ_B/L_B.
2. Using equation 3, obtain δ_A/L_A.
3. Obtain F_B/A_B and F_A/A_A from Figure (*b*) as explained in equation 4.
4. Check whether these values satisfy equation 5.
5. If equation 5 is not satisfied, make a new guess for δ_B/L_B and obtain new values for δ_A/L_A, F_A/A_A and F_B/A_B.

6. Proceeding in this way, we find the points *a* and *b* in Figure (*b*). From these points,

$$\frac{\delta_B}{L_B} = 0.0005 \quad \Rightarrow \quad \frac{F_B}{A_B} = 41MN/m^2 \quad \Rightarrow \quad F_B = 53.3kN$$

$$\frac{\delta_A}{L_A} = 0.001 \quad \Rightarrow \quad \frac{F_A}{A_A} = 74MN/m^2 \quad \Rightarrow \quad F_A = 96.2kN$$

$$\delta_A = \delta_B = 1.3\text{mm}$$

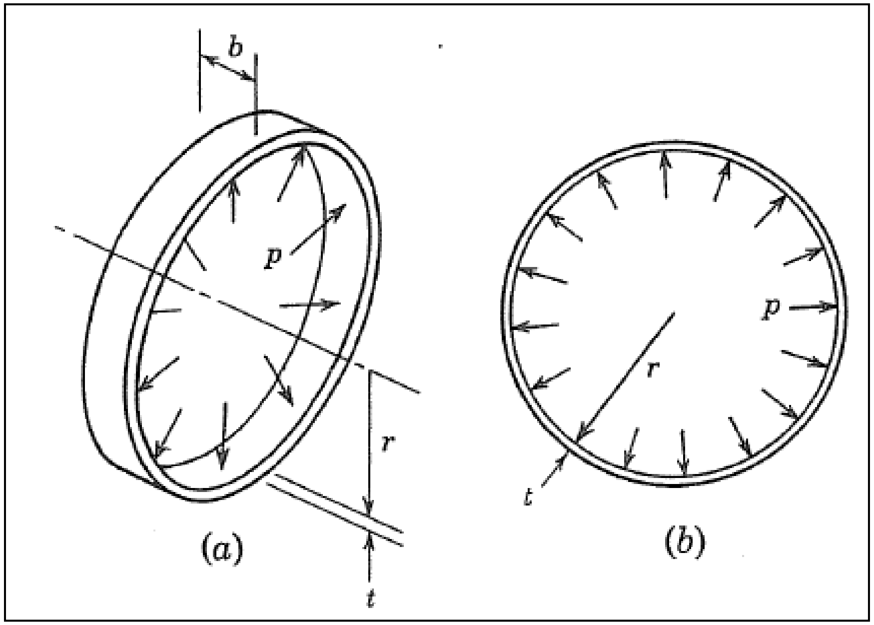
Substituting this value for F_B in the equation 1(A), we obtain the required location of the roller

$$c = 0.355\text{m}$$

Analysis of thin ring

Example

A thin ring of internal radius *r*, thickness *t*, and width *b* is subjected to a uniform pressure *p* over the entire internal surface, as shown in figure below. We would like to determine the forces in the ring and deformation of the ring due to the internal pressure.



Introduction to Mechanics of deformable bodies

1. Force equilibrium

F_R must be zero because Newton's third law and symmetry argument, both together are not satisfied at a time.

Let's consider upper half of the ring

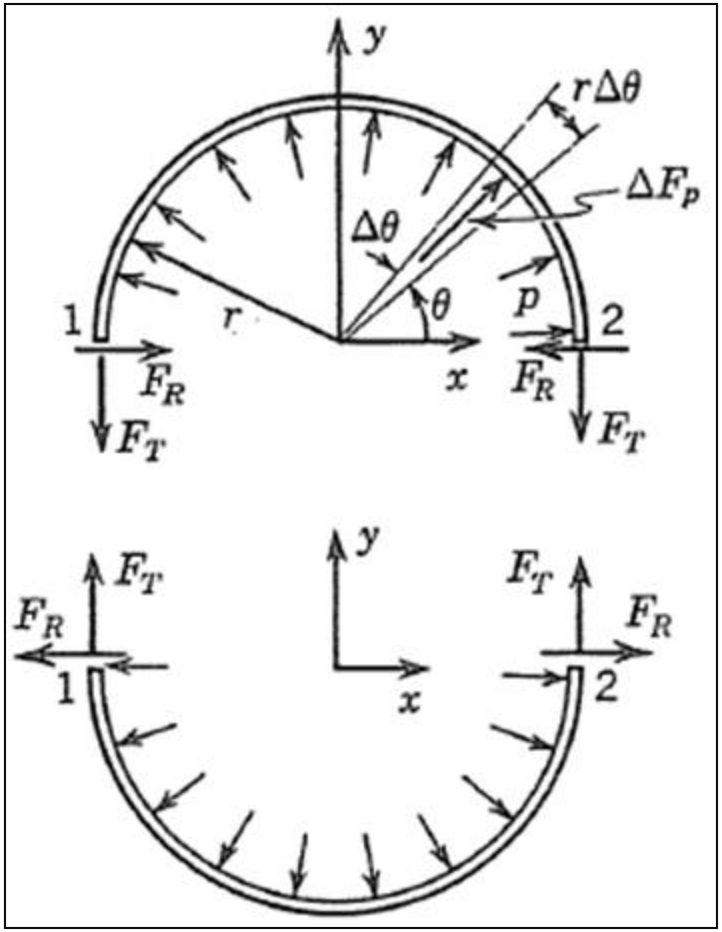
$$\Delta F_p = p[b(r\Delta\theta)] \quad \text{--- (a)}$$

$$\Delta F_y = \Delta F_p \sin\theta = p[b(r\Delta\theta)]\sin\theta \quad \text{-----(b)}$$

$$\sum F_y = \int_{\theta=0}^{\theta=\pi} pbr\sin(\theta)d\theta - 2F_T = 0 \quad \text{---- (c)}$$

$$F_T = prb \quad \text{---- (d)}$$

$$\sum F_y = p(2rb) - 2F_T = 0 \quad \text{---- (e)}$$



Introduction to Mechanics of deformable bodies

2. Force-deformation relation

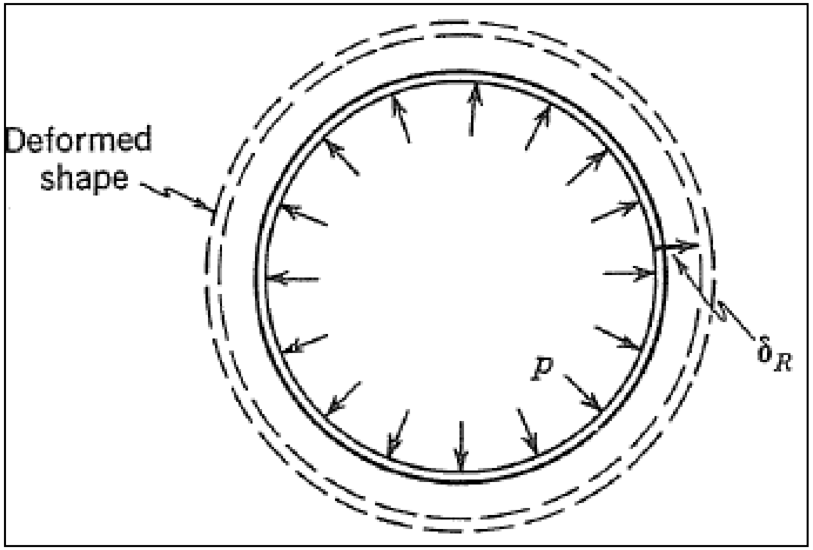
$$\delta_T = \frac{F_T[2\pi(r + t/2)]}{(bt)E} = \frac{2\pi pr^2}{tE}(1+\frac{t}{2r}) \quad \text{----- (f)}$$

3. Geometric compatibility

$$\delta_R = \frac{\delta_T}{2\pi} \quad \text{--- (g)}$$

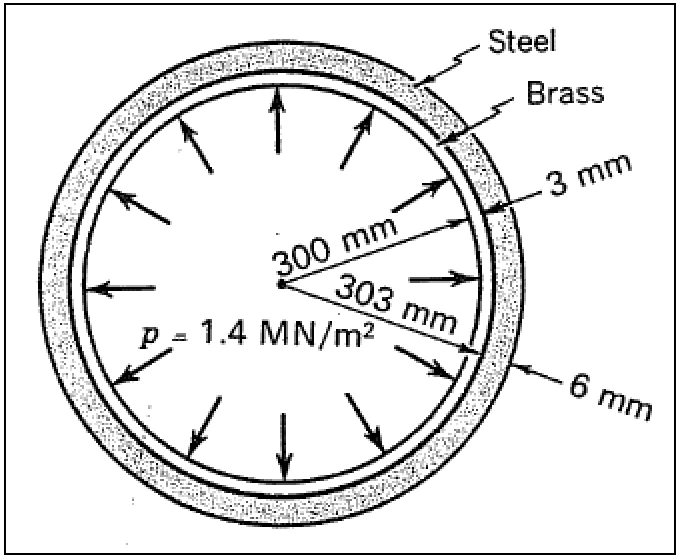
$$\delta_R = \frac{pr^2}{tE}(1+\frac{t}{2r}) \quad \text{---- (h)}$$

$$\delta_R = \frac{pr^2}{tE} \quad \text{----- (i)}$$



Problem

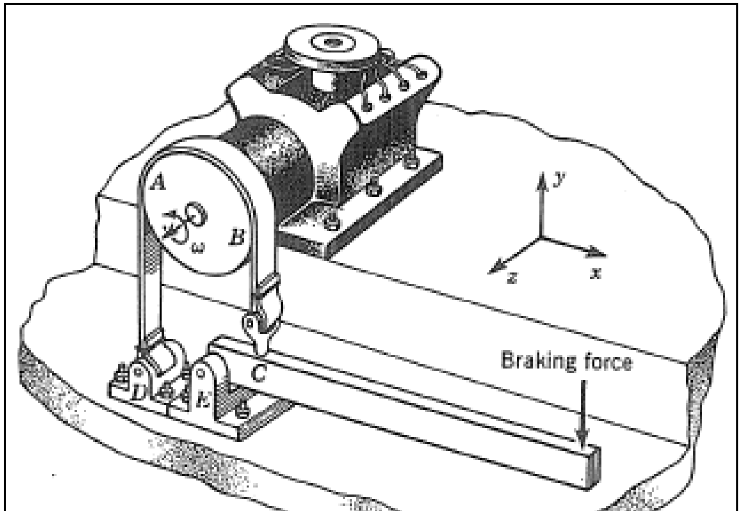
A composite hoop consists of a brass hoop of 300-mm internal radius and 3-mm thickness, and a steel hoop of 303-mm internal radius and 6-mm radial thickness. Both hoops are 6-mm thick normal to the hoop. If radial pressure of 1.4 MN/m^2 is put in the brass hoop, estimate the tangential forces in the brass and steel hoops.



Analysis of flexible drives

Example

In a test on an engine, a braking force is supplied through a lever arm *EF* to a steel brake *CBAD* which is in contact with half the circumference of a 600mm diameter flywheel. The brake band is 1.6mm thick and 50mm wide and is lined with a relatively soft material which has a kinetic coefficient of friction of f = 0.4 with respect to the rotating flywheel. The operator wishes to predict how much elongation there will be in the section *AB* of the brake band when the braking force is such that there is a tension of 40kN in the section *BC* of the band.



(a)

Introduction to Mechanics of deformable bodies

1. Force equilibrium

$$\sum F_r = \Delta N - T\sin\frac{\Delta\theta}{2} - (T+\Delta T)\sin\frac{\Delta\theta}{2} = 0 \quad \text{--- 1}$$

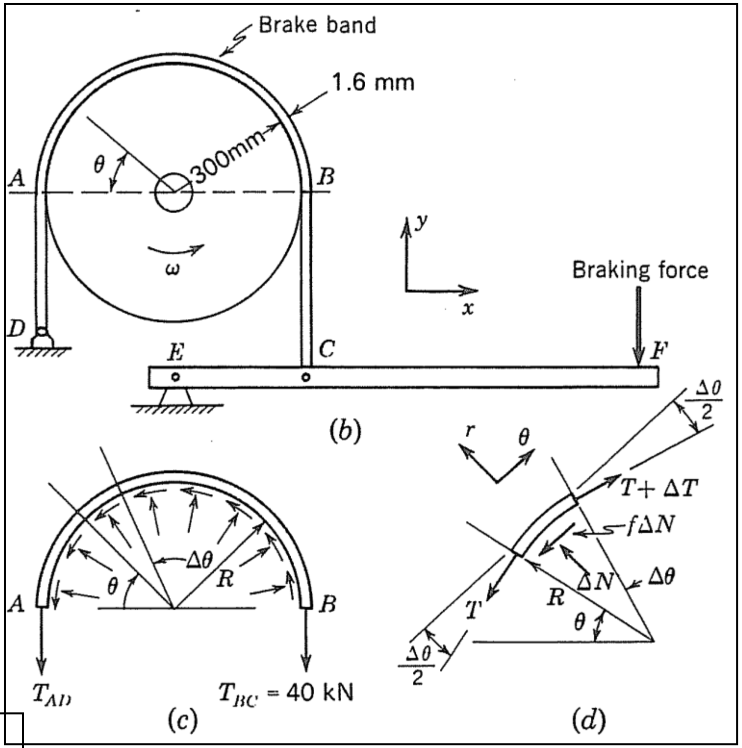
$$\Delta N - T\frac{\Delta\theta}{2} - (T+\Delta T)\frac{\Delta\theta}{2} = 0 \quad \Rightarrow \quad \Delta N = T\Delta\theta \quad \text{--- 2}$$

$$\sum F_\theta = (T+\Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - f\Delta N = 0 \quad \text{--- 3}$$

$$(T+\Delta T) - T - f\Delta N = 0 \quad \Rightarrow \quad \Delta T = f\Delta N \quad \text{--- 4}$$

From 2 & 4

$$\Delta T = fT\Delta\theta \quad \Rightarrow \quad \frac{\Delta T}{T} = f\Delta\theta \quad \Rightarrow \quad \frac{dT}{T} = fd\theta \quad \text{--- 5}$$



At $\theta = 0$; $T = T_{AD}$ and $\theta = \pi$; $T = T_{BC}$

$$\int_{T_{AD}}^{T_{BC}} \frac{dT}{T} = f \int_0^{\pi} d\theta \quad \text{--- 6}$$

$$\frac{T_{BC}}{T_{AD}} = e^{f\pi} \quad \text{--- 7}$$

where e is the base of natural logarithms. T_{AD} can be calculated by substituting T_{BC} = 40kN and f = 0.4 in equation 7.

Suppose, if we assume T_1 is tension in tight side; T_2 is tension in slack side and θ is angle of contact in radians, then equation 7 can be written in the following form

$$\frac{T_1}{T_2} = e^{f\theta} \quad \text{--- 8}$$

2. Force-deformation relation

Consider the deflection of an small element of length $R\Delta\theta$ and cross-section area A

$$\Delta\delta = \frac{TR\,\Delta\theta}{AE} \quad \Rightarrow \quad d\delta = \frac{TR\,d\theta}{AE} \qquad \text{--- 9}$$

3. Geometric compatibility

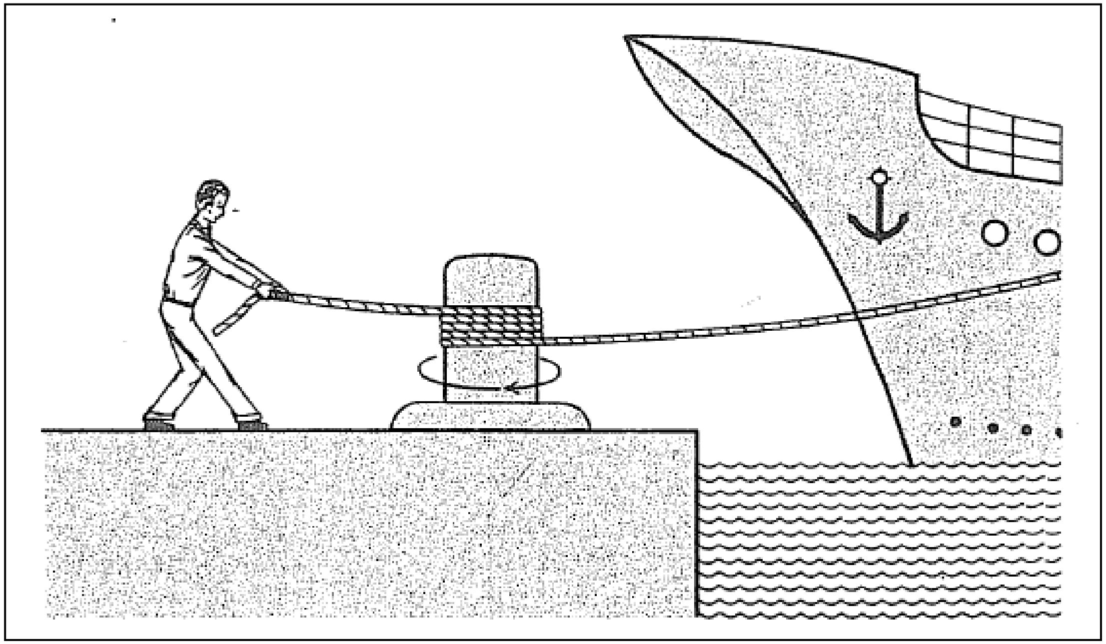
Total elongation of the brake band from A to B, δ_{AB}, is the sum of the tangential elongations of small elements of length $R\Delta\theta$.

$$\delta_{AB} = \int_{\theta=0}^{\theta=\pi} d\delta = \int_{\theta=0}^{\theta=\pi} \frac{TR\,d\theta}{AE} \qquad \text{--- 10}$$

$$\delta_{AB} = \frac{T_{AD}R}{AE}\int_{0}^{\pi} e^{f\theta}d\theta = \frac{T_{AD}R}{AEf}(e^{f\pi}-1) \qquad \text{--- 11}$$

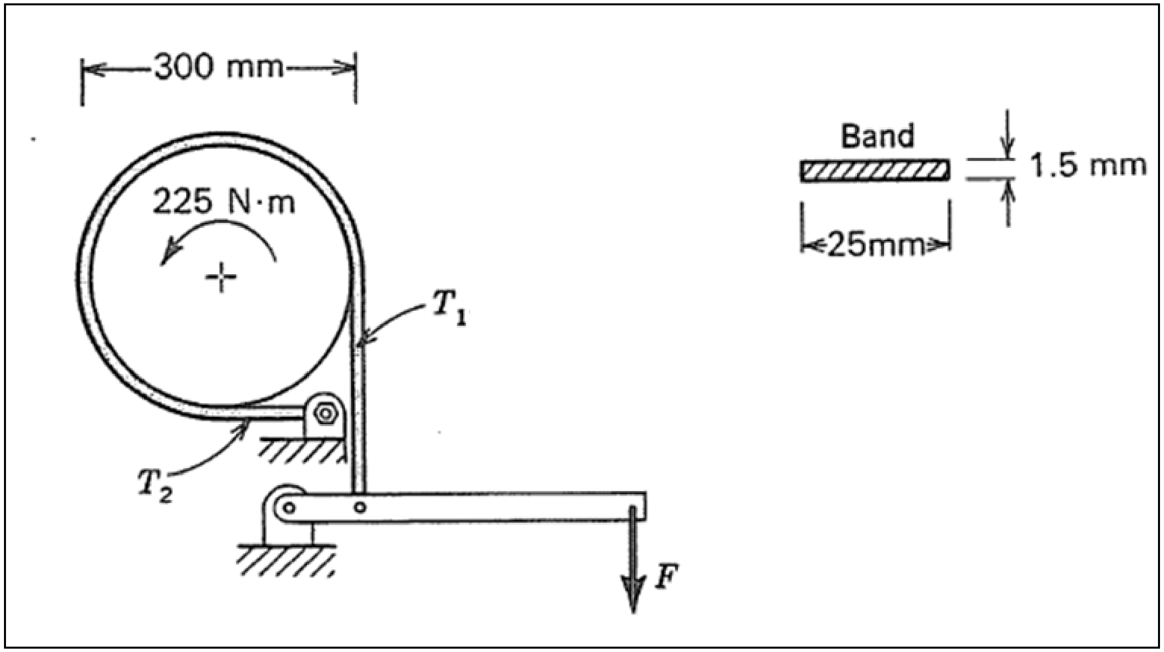
Problem

A hawser from a ship is wrapped four times around a rotating capstan as shown in the figure. The dockworker pulls with a force of 200 N. What is the maximum force the man can exert on the boat if the coefficient of friction between the capstan and hawser is 0.3?



Problem

A brake is designed as shown. A 25 x 1.5 mm steel band restrains the wheel from turning when a 225 N-m torque is applied. The friction coefficient is 0.4. Find the tensions T_1 and T_2 that just keep the wheel from rotating.

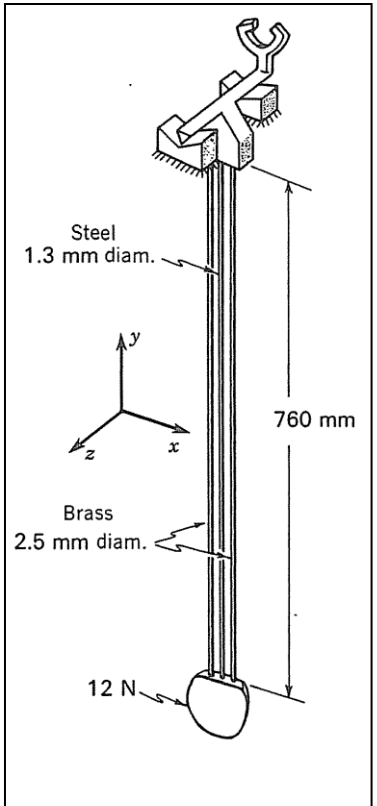


Statically indeterminate situations

These are the situations in which deformations must be considered to determine forces.

Example

Figure shows the pendulum of a clock which has a12N weight suspended by three rods of 760mm length. Two of the rods are made of brass and the third of steel. We wish to know how much of the 12N suspended weight is carried by each rod. Take E_S = 200GPa and E_B = 100 Gpa.



(a)

Solution

1. Force equilibrium:

$$\sum F_y = 12 - F_S - 2F_B = 0 \quad \text{--- 1}$$

2. Geometric compatibility:

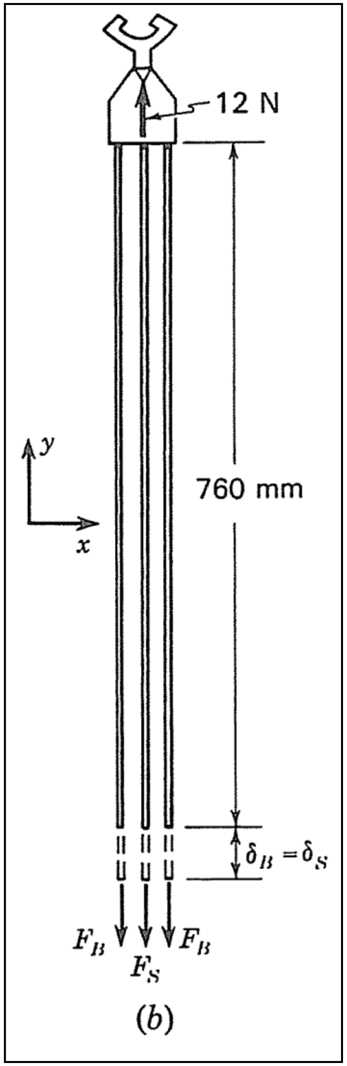
$$\delta_S = \delta_B \quad \text{--- 2}$$

3. Force-deformation relation:

$$\delta_S = \frac{F_S L_S}{A_S E_S} \quad \text{and} \quad \delta_B = \frac{F_B L_B}{A_B E_B} \quad \text{--- 3}$$

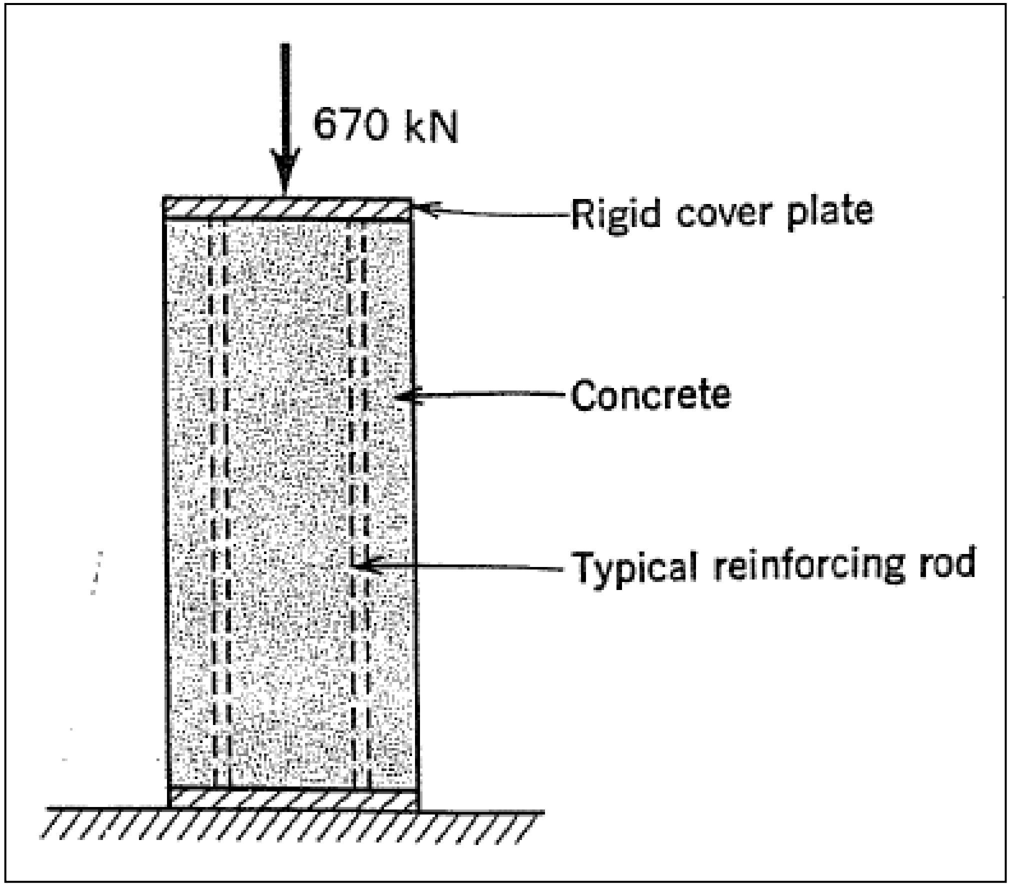
From equations 1 to 3

$$F_S = 2.55N \quad \text{and} \quad F_B = 4.72N$$



Problem

A square reinforce-concrete pier 0.3 × 0.3m in cross section and 1.2m high is loaded as shown in the figure. The concrete is strengthened by the addition of eight vertical 25 × 25mm square steel reinforcing bars placed symmetrically about the vertical axis of the pier. Find the stress (force per unit area) in the steel and concrete and the deflection. Take E_C = 17GPa and E_S = 200GPa.



ELASTIC ENERGY; CASTIGLIANO'S THEOREM

Conservative systems:

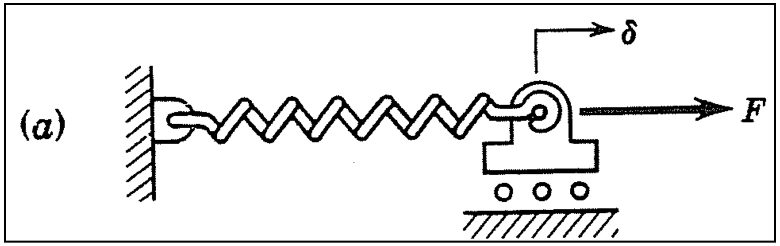
When work is done by an external force on certain systems, their internal geometric states are altered in such a way that they have the potential to give back equal amounts of work whenever they are returned to their original configurations.

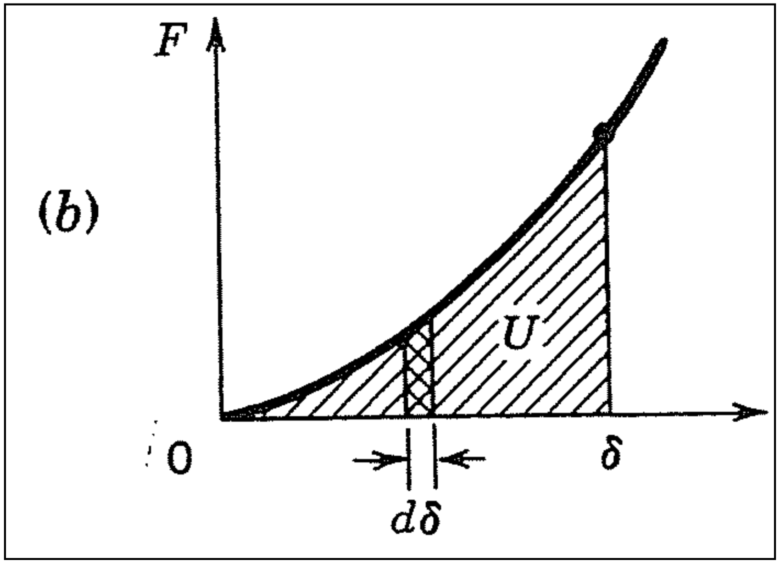
e.g. Elastic spring

Strain Energy:

Consider the elastic, but not necessarily linear, spring in Fig (*a*). Let the spring undergo a gradual elongation process during which the external force *F* remains in equilibrium with the internal tension. The potential or strain energy *U* associated with an elongation δ is defined to be the work done by *F* in this process

$$U = \int_0^{\delta} F d\delta$$

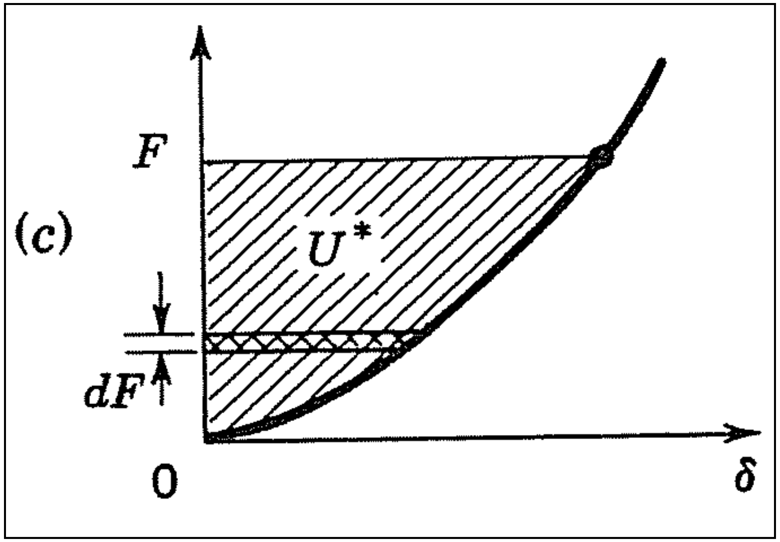




Complementary Energy:

When the point of application of a variable force *F* undergoes a displacement δ, the complementary work is done on the system.

When complementary work is done on certain systems, their internal force states are altered in such a way that they are capable of giving up equal amounts of complementary work when they are returned to their original force states. Under these circumstances the complementary work done on such a system is said to be stored as complementary energy.



$$U^* = \int_0^{\delta} \delta dF$$

Introduction to Mechanics of deformable bodies

Castigliano's theorem:

It states that if the total complementary energy U^* of a loaded elastic system is expressed in terms of the loads, the in-line deflection at any particular loading point is obtained by differentiating U^* with respect to the load at that point.

$$\delta = \frac{\partial U^*}{\partial P}$$

- For nonlinear systems, $U \neq U^*$
- For linear systems, $U = U^*$
- Strain Energy of linear spring is $U = \frac{1}{2} F\delta = \frac{1}{2} k\delta^2 = \frac{1}{2}\frac{F^2}{k}$

Introduction to Mechanics of deformable bodies

- This chapter, we will consider application of castigliano's theorem only for linear elastic systems.
- For linear system, complementary energy is equal to the strain energy.
- Let us consider a system, consists of *N* elastic elements. External force P_i acting at point A_i. We wish to know the deflection of any point A_i.

Total strain energy of the system is

$$U = \sum_{i=1}^{n} \frac{1}{2} \frac{F_i^2}{k_i}$$

where F_i is the internal force induced in elastic member and it should be expressed in terms of P_i

Introduction to Mechanics of deformable bodies

- Deformation (δ_i), at point A_i in the direction of P_i is given by

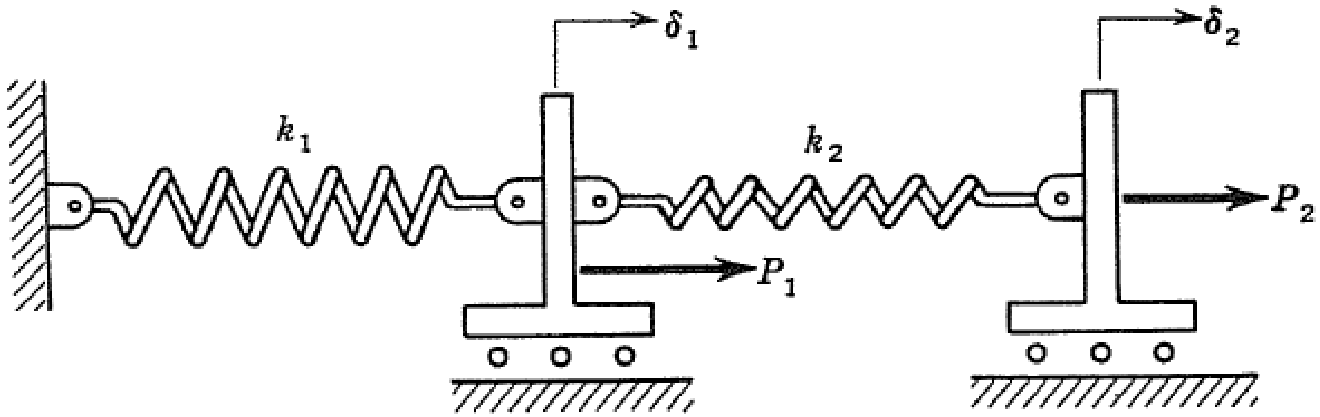
$$\delta_i = \frac{\partial U}{\partial P_i}$$

Important

- If, we may wish to know deflection (may be in horizontal or vertical or in any other direction) at a point where external force is zero .
- In such case a fictitious force Q is to be considered at that point.
- Express internal forces in terms of Q.
- Deflection at that point in the direction of Q is given by $\partial U/\partial Q$ and setting Q = 0.

Example

Consider the system of two springs shown in Figure and determine the deflections using Castigliano's theorem.



Solution

From equilibrium requirements, the spring forces (F_1 and F_2) are expressed as

$$F_1 = P_1 + P_2 \qquad \& \qquad F_2 = P_2$$

The total elastic energy is given by

$$U = U_1 + U_2 = \frac{(P_1 + P_2)^2}{2k_1} + \frac{P_2{}^2}{2k_2}$$

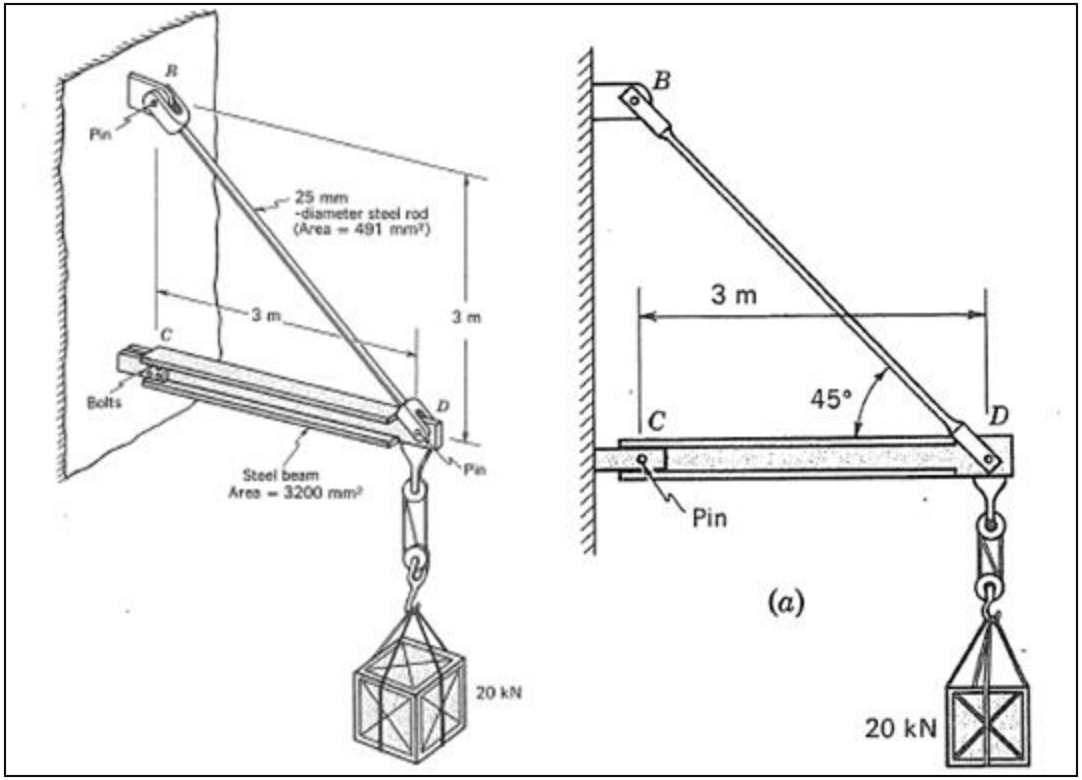
The deflections (δ_1 and δ_2) are

$$\delta_1 = \frac{\partial U}{\partial P_1} = \frac{P_1 + P_2}{k_1} \quad \text{and} \quad \delta_2 = \frac{\partial U}{\partial P_2} = \frac{P_1 + P_2}{k_1} + \frac{P_2}{k_2}$$

Introduction to Mechanics of deformable bodies

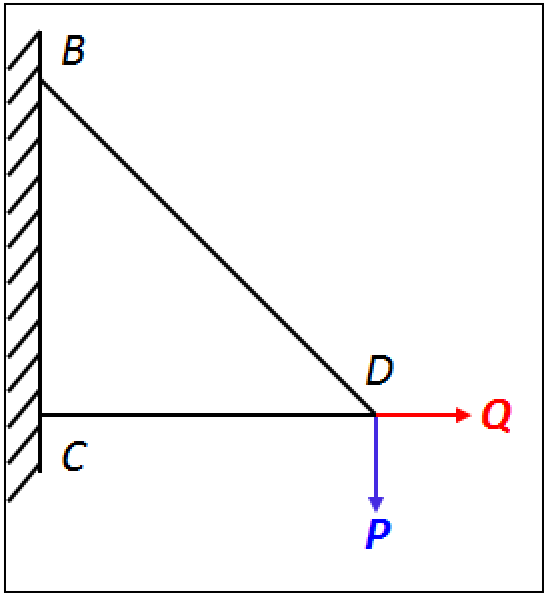
Example

Figure shows a triangular frame supporting a load of 20kN. Using Castigliano's theorem, determine vertical and horizontal deflection of point *D*.



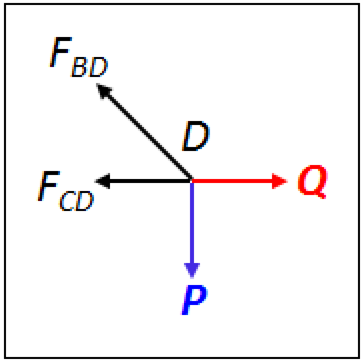
Solution

- Vertical force say ***P*** is acting at point *D* (***P*** = 20kN).
- Horizontal force is zero at point *D*.
- Lets consider a fictitious force ***Q*** acting at point *D* in horizontal direction as shown in figure.
- Express F_{BD} and F_{CD} in terms of P and Q.



Introduction to Mechanics of deformable bodies

From FBD of joint *D*,



$$\sum F_Y = 0 = F_{BD}\sin 45 - P \quad \Rightarrow \quad F_{BD} = \sqrt{2}P$$

$$\sum F_X = 0 = Q - F_{CD} - F_{BD}\cos 45 \quad \Rightarrow \quad F_{CD} = Q - P$$

Total strain energy, *U* of the system is

$$U = U_{BD} + U_{CD} = \frac{F_{BD}^{\;2}}{2k_{BD}} + \frac{F_{CD}^{\;2}}{2k_{CD}} = \frac{2P^2}{2k_{BD}} + \frac{(Q-P)^2}{2k_{CD}}$$

where k_{BD} and k_{CD} are stiffness of bar *BD* and *CD*

$$k_{BD} = \frac{A_{BD}\times E}{L_{BD}} = \frac{491\times 205\times 10^3}{3\sqrt{2}\times 10^3} = 23.72\times 10^3\ N/mm$$

$$k_{CD} = \frac{A_{CD}\times E}{L_{CD}} = \frac{3200\times 205\times 10^3}{3\times 10^3} = 218.67\times 10^3\ N/mm$$

Vertical deflection of point *D*,

$$\delta_{VD} = \frac{\partial U}{\partial P} = \frac{4P}{2k_{BD}} + \frac{2(P-Q)}{2k_{CD}}$$

Setting *P* = 20kN and *Q* = 0

$$\delta_{VD} = \frac{4\times 20\times 10^3}{2\times 23.73\times 10^3} + \frac{2\times\left(20\times 10^3 - 0\right)}{2\times 218.66\times 10^3} = 1.77mm$$

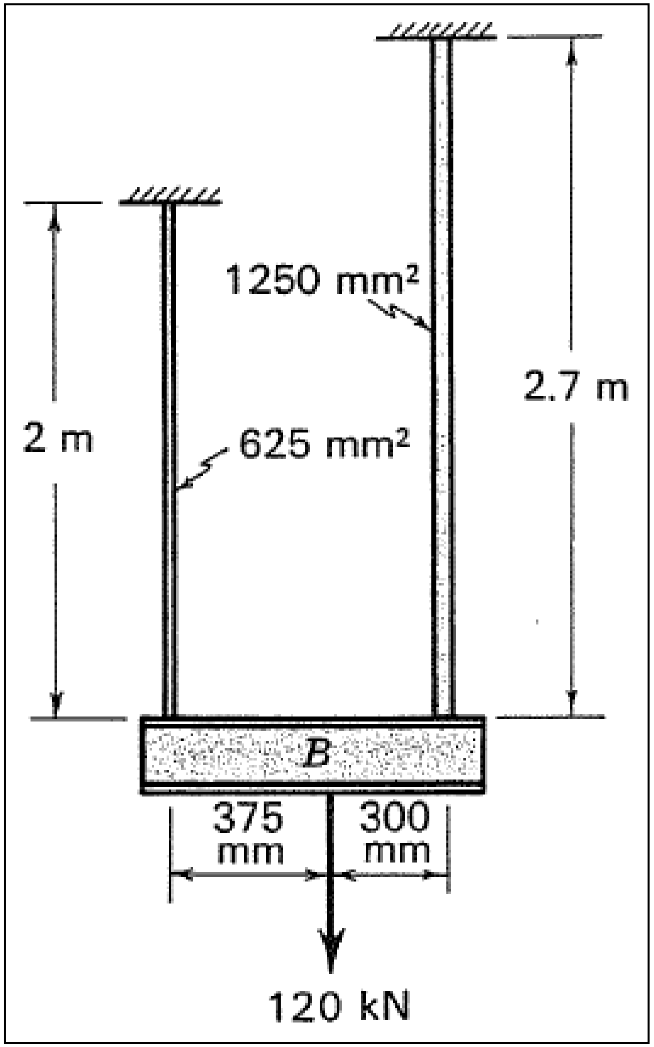
horizontal deflection of point *D*,

$$\delta_{HD} = \frac{\partial U}{\partial Q} = 0 + \frac{2(Q-P)}{2k_{CD}} = \frac{2\times\left(0 - 20\times 10^3\right)}{2\times 218.66\times 10^3} = -0.091mm$$

-ve sign indicates that the horizontal deflection of point *D* is in opposite to the direction of ***Q*** i.e. rod *CD* gets compressed.

Problem

A very stiff horizontal member is supported by two vertical steel rods of different cross section area and length. If a vertical load of 120kN is applied to the horizontal beam at point *B*, estimate the vertical deflection of the point *B*.



innovate achieve lead

References

1. Introduction to Mechanics of Solids by S. H. Crandall et al (In SI units), McGraw-Hill